Using Answer Set Programming for Representing and Reasoning with Preferences and Uncertainty in Dynamic Domains

Ravi Palla¹,², Dan Tecuci¹, Vinay Shet¹, Mathaeus Dejori¹

¹Siemens Corporate Research, Princeton, NJ, USA
{dan.tecuci,vinay.shet,mathaeus.dejori}@siemens.com
²School of Computing, Informatics, and Decision Systems Engineering, Arizona State University, Tempe, AZ, USA
ravi.palla@asu.edu

Abstract. We investigate the use of Answer Set Programming for representing and reasoning with preferences and uncertainty in dynamic domains. We consider a few sentences from the medical guidelines for treatment of STEMI (ST Elevation Myocardial Infarction), represent them in ASP, and show how certain questions can be answered using existing answer set solvers.

1 Introduction

Diagnosis and treatment of diseases or disorders is a complex process, often requiring physicians to consider multiple causes with relative likelihoods and uncertain effects. Any biomedical Question-Answering (QA) system that is required to assist physicians in these tasks needs to be able to encode and reason with such complex knowledge. Consider the sentence “Heart failure is usually due to myocardial damage but may also be the consequence of arrhythmia or mechanical complications such as mitral regurgitation or ventricular septal defect”. This sentence indicates that heart failure has more than one cause and that usually, the cause is myocardial damage. Further, the statement also indicates that myocardial damage, arrhythmia, mitral regurgitation and ventricular septal defect “may” cause heart failure. We expect a QA system to be able to recognize myocardial damage as the most likely cause of heart failure, but, at the same time, be able to return other causes as well. Furthermore, we expect the system to be able to predict that myocardial damage, arrhythmia, mitral regurgitation and ventricular septal defect “might” cause heart failure. The system can accomplish the former by encoding some kind of preference between the causes, and the latter by encoding the uncertainty in the effects of the causes.

In this paper, we investigate the use of Answer Set Programming (ASP) [1; 2; 3] for representing and reasoning with preferences and uncertainty in dynamic domains. We consider a few sentences from the medical guidelines for treatment of STEMI (ST Elevation Myocardial Infarction), represent them in ASP, and show how certain questions can be answered using existing answer set solvers.

2 Sample Sentences and Questions

We consider the following two sets of sentences in this paper:
Most cases of STEMI are caused by an occlusion of a major coronary artery.
Coronary occlusion and reduction in coronary blood flow are usually due to physical disruption of an atherosclerotic plaque with subsequent formation of an occluding thrombus.
Less commonly, a thrombus may form from a superficial erosion of the endothelial surface.

Heart failure is usually due to myocardial damage but may also be the consequence of arrhythmia or mechanical complications such as mitral regurgitation or ventricular septal defect.
A life-threatening arrhythmia, such as ventricular tachycardia, ventricular fibrillation, and total atrio-ventricular block, may be the first manifestation of ischaemia and requires immediate correction.
These arrhythmias may cause many of the reported sudden cardiac deaths in patients with acute ischaemic syndromes.

3 Representing Preferences and Uncertainty

For our representation, we consider extended logic programs \[4\] with optional labels and choice formulas. More formally, an \(L\)-\(C\) program, where \(L\) stands for Label and \(C\) stands for Choice, is a finite set of rules of the form

\[
[label : ] L \leftarrow L_1, \ldots, L_n, \text{not } L_{n+1}, \ldots, \text{not } L_m
\]  

or

\[
[label : ] \{L\} \leftarrow L_1, \ldots, L_n, \text{not } L_{n+1}, \ldots, \text{not } L_m
\]

where “\([label : ]\)” represents the optional label, \(m \geq n \geq 0\), and \(L\) and each \(L_i\) are literals or \(\top\) or \(\bot\). For any rule of the form (1) (resp. (2)), \(L\) (resp. \(\{L\}\)) is called the head of the rule and \(L_1, \ldots, L_n, \text{not } L_{n+1}, \ldots, \text{not } L_m\) is called the body of the rule. A rule of the form (1) where \(L\) is \(\bot\) is called a constraint and a rule with empty body \((m = n = 0)\) is called a fact. A rule of the form (2) is called a choice rule and is used for representing uncertainty. Preferences are represented in our language using the prefer
predicate, similar to CR-Prolog [5]. For example, \texttt{prefer(label1, label2)} represents that the rule with label \texttt{label1} should be preferred over the rule with label \texttt{label2}.\footnote{For simplicity, we assume that the label of a rule is unique.} As usual, variables are understood as place-holders, to be replaced by the elements in the universe.

In order to answer questions, \(L\)-\(C\) programs are first translated to ASP. The translation we employ is similar to that used in CR-Prolog for forming the \emph{hard reduct}. Given an \(L\)-\(C\) program \(\Pi\), by \(\Pi_t\) we represent the program obtained from \(\Pi\) by

\begin{itemize}
  \item replacing every labeled rule of the form (1) or (2) by
    \begin{align*}
      L & \leftarrow L_1, \ldots, L_n, \text{not } L_{n+1}, \ldots, \text{not } L_m, \text{apply}(\text{label})
    \end{align*}
    \tag{3}
  \\
  \text{and}
  \begin{align*}
    \{L\} & \leftarrow L_1, \ldots, L_n, \text{not } L_{n+1}, \ldots, \text{not } L_m, \text{apply}(\text{label})
  \end{align*}
  \tag{4}
  \\
  \text{respectively.}
  \item adding the rule
    \begin{align*}
      \{\text{apply}(\text{label})\}
    \end{align*}
    \tag{5}
  \end{itemize}

for every labeled rule of the form (1) or (2).

- adding the rules
  \begin{align*}
    \text{is}\_\text{preferred}(R_1, R_2) & \leftarrow \text{prefer}(R_1, R_2) \tag{6} \\
    \text{is}\_\text{preferred}(R_1, R_2) & \leftarrow \text{prefer}(R_1, R_3), \text{is}\_\text{preferred}(R_3, R_2) \tag{7} \\
    & \leftarrow \text{is}\_\text{preferred}(R, R) \tag{8} \\
    & \leftarrow \text{apply}(R_1), \text{apply}(R_2), \text{is}\_\text{preferred}(R_1, R_2) \tag{9}
  \end{align*}

where \(R, R_1, R_2, R_3\) are variables that range over the labels of the rules.

Here, we assume that \text{apply} and \text{is}\_\text{preferred} do not occur in \(\Pi\). The rule (5) denotes an arbitrary choice for application of the rule with label \text{label}. The rules (6) and (7) represent the transitive closure of \text{prefer}, and the constraint (8) enforces the condition that preferences are non-reflexive. The constraint (9) disallows the combined application of two rules if one is preferred over the other.

If \(X\) and \(Y\) are two answer sets of \(\Pi_t\), we say that \(X\) is \text{better than} \(Y\) if there exist atoms \text{apply}(r_1) \in X \text{ and } \text{apply}(r_2) \in Y \text{ such that } \text{is}\_\text{preferred}(r_1, r_2) \in X \cap Y\). We say that \(X\) is \text{strictly better than} \(Y\) if \(X\) is better than \(Y\) and it is not the case that \(Y\) is better than \(X\). By \text{body}(r), we denote the body of the rule with label \(r\). If \text{body}(r) is \(L_1, \ldots, L_n, \text{not } L_{n+1}, \ldots, \text{not } L_m\), we say that a set of literals \(X\) satisfies \text{body}(r), denoted by \(X \models \text{body}(r)\), if \(L_i \in X\) for \(i = 1 \ldots n\) and \(L_j \notin X\) for \(j = n + 1 \ldots m\).

An answer set \(X\) of \(\Pi_t\) is \text{non-redundant} if \(X \models \text{body}(r)\) for all \text{apply}(r) \in X. Finally, we say that a consistent set of literals \(X\) is a \text{preferred answer set} of \(\Pi_t\) if it is non-redundant and there is no non-redundant answer set of \(\Pi_t\) that is strictly better than \(X\). Note that preferred answer sets are not meant to provide semantics for \(L\)-\(C\) programs, but are rather used to narrow the search for the answer.
Consider the following program $\Pi$:

\[
\begin{align*}
    r_1 &: p \leftarrow q & r_3 &: r \leftarrow s \\
    r_2 &: p \leftarrow r & r_4 &: q \leftarrow s
\end{align*}
\]  

(10)

Rules $r_1, r_2$ represent that $p$ is caused by either $q$ or $r$, and the preference relation between $r_1$ and $r_2$ represents that $p$ is usually caused by $q$. Similarly, rules $r_3, r_4$ represent that $s$ causes either $r$ or $q$, and the preference relation between the two rules represents that it usually causes $r$. $\Pi'$ consists of the rules (6)–(9), and the following rules:

\[
\begin{align*}
    p &\leftarrow q, \text{apply}(r_1) & r &\leftarrow s, \text{apply}(r_3) \\
    p &\leftarrow r, \text{apply}(r_2) & q &\leftarrow s, \text{apply}(r_4) \\
    \{\text{apply}(r_1)\} &\leftarrow \{\text{apply}(r_3)\} \\
    \{\text{apply}(r_2)\} &\leftarrow \{\text{apply}(r_4)\} \\
    \text{prefer}(r_1, r_2) &\leftarrow \text{prefer}(r_3, r_4)
\end{align*}
\]

Consider the following query $Q$:

\[
\begin{align*}
    \{s\} \\
    \leftarrow \text{not } p,
\end{align*}
\]

that essentially asks for the likely cause / explanation for $p$. $(II \cup Q)'$ has 2 answer sets:

\[
\begin{align*}
    \{p, r, s, \text{prefer}(r_1, r_2), \text{prefer}(r_3, r_4), \text{apply}(r_3), \text{apply}(r_2), \\
    \text{is_preferred}(r_1, r_2), \text{is_preferred}(r_3, r_4)\}
\end{align*}
\]

and

\[
\begin{align*}
    \{p, q, s, \text{prefer}(r_1, r_2), \text{prefer}(r_3, r_4), \text{apply}(r_4), \text{apply}(r_1), \\
    \text{is_preferred}(r_1, r_2), \text{is_preferred}(r_3, r_4)\}
\end{align*}
\]

Both these are preferred answer sets, and so we conclude that $p$ has two likely causes $\{r, s\}$ and $\{q, s\}$.

It is important to note that preferred answer sets are not always intuitive. For example, consider the program (10) $\cup \{s \leftarrow\}$. One of the preferred answer sets is

\[
\begin{align*}
    \{s, \text{prefer}(r_1, r_2), \text{prefer}(r_3, r_4), \text{is_preferred}(r_1, r_2), \text{is_preferred}(r_3, r_4)\}.
\end{align*}
\]

which indicates that neither $r_3$ nor $r_4$ is applied. Therefore, such answers sets need to be discarded while answering questions.

4 Question-Answering

4.1 Answering Questions Related to the First Set of Sentences

Consider the first set of sentences shown in Section 2. A simplified representation of these sentences in our language is given by the following program $II$. We use $h(f, t)$
(resp. \(\neg h(f, t)\)) to represent that fluent \(f\) is true (resp. false) at time \(t\).

\[
\begin{align*}
    r_{\text{stemi}}(T) : \\
    \{h(\text{stemi}, T + 1)\} & \leftarrow h(\text{coronary occlusion}, T) \\
    r_{\text{cor occ}}(T) : \\
    \{h(\text{coronary occlusion}, T + 1)\} & \leftarrow h(\text{occluding thrombus}, T) \\
    r_{\text{cor bldflw red}}(T) : \\
    \{h(\text{coronary bloodflow reduction}, T + 1)\} & \leftarrow h(\text{occluding thrombus}, T) \\
    r_{\text{thrombus}}._1(T) : \\
    h(\text{occluding thrombus}, T + 1) & \leftarrow h(\text{disruption plaque}, T) \\
    r_{\text{thrombus}}._2(T) : \\
    \{h(\text{occluding thrombus}, T + 1)\} & \leftarrow h(\text{erosion endo surface}, T) \\
    \textit{prefer}(r_{\text{thrombus}}._1(T), r_{\text{thrombus}}._2(T))
\end{align*}
\]

The rule with label \(r_{\text{stemi}}(T)\) represents that STEMI may be caused by a coronary occlusion. Since there is no other cause of STEMI specified in the sentences, no preference relation is specified for this rule. The rules with labels \(r_{\text{cor occ}}(T)\) and \(r_{\text{cor bldflw red}}(T)\) represent the knowledge that coronary occlusion and reduction in coronary blood flow might be caused by an occluding thrombus. The rule \(r_{\text{thrombus}}._1(T)\) represents the knowledge that disruption of plaque causes formation of an occluding thrombus, and \(r_{\text{thrombus}}._2(T)\) represents that erosion of the endothelial surface may cause formation of an occluding thrombus. Finally, the preference relation between \(r_{\text{thrombus}}._1(T)\) and \(r_{\text{thrombus}}._2(T)\) represents that formation of a thrombus due to erosion of the endothelial surface is less common than formation of a thrombus due to disruption of plaque. Note that we do not identify the implicit events such as the ones involved in formation of a thrombus since we would like to keep the representation as close to the sentences as possible.

Since we represent time, we need to consider the commonsense law of inertia. In order to represent inertia, we use the following set of rules \(I: \)

\[
\begin{align*}
    h(F, T + 1) & \leftarrow h(F, T), \neg h(F, T + 1) \\
    \neg h(F, T + 1) & \leftarrow \neg h(F, T), \neg h(F, T + 1).
\end{align*}
\]

Here, \(F\) ranges over all the fluents in the representation. The rules represent the knowledge that a fluent retains its earlier value unless there is a cause for a change.

The first question asks for the most likely cause of STEMI. To get the immediate causes of STEMI, we use the following query \(Q_1: \)

\[
\begin{align*}
    \{h(\text{stemi}, 0)\} \\
    \{\neg h(\text{stemi}, 0)\} \\
    \leftarrow h(\text{stemi}, 0) \\
    \leftarrow \neg h(\text{stemi}, \text{maxstep})
\end{align*}
\]
The first two rules represent that all the fluents are initially exogenous. Since we want to find the cause of STEMI, we use the third rule to represent that STEMI is initially not true. This rule discards all the sets in which STEMI is initially true. The last rule discards all the sets in which \( \text{stem}i \) is not true at the maximum timepoint of interest, denoted by \( \text{maxstep} \). To get the immediate causes, we set \( \text{maxstep} \) to 1.

For \( \text{maxstep} = 1 \), every preferred answer set of \( (\Pi \cup I \cup Q_1)^t \) represents a likely immediate cause of STEMI. For this example, every preferred answer set contains \( h(\text{coronary occlusion}, 0) \), indicating that the most likely cause of STEMI is coronary occlusion. Note that the preferred answer sets might suggest that other fluents might also be true at timepoint 0. However, we can pick the answer by restricting attention to the bodies of those rules \( r \) for which \( \text{apply}(r) \) belongs to the preferred answer set. For example, one of the preferred answer sets of \( (\Pi \cup I \cup Q_1)^t \) for \( \text{maxstep} = 1 \) contains \( h(\text{coronary blood flow reduction}, 0) \) but we can discard this atom since it does not occur in the body of any rule.

In order to actually compute the answer, one can run \( (\Pi \cup I \cup Q_1)^t \) (with \( \text{maxstep} = 1 \)) using an existing answer set solver like CLINGO\(^2\) and then pick the preferred answer sets.

The query \( Q_1 \) was used to get the immediate cause of STEMI. However, we might need to retrieve the sequence of causes. This can be done by using the following query \( Q_2 \):

\[
\begin{align*}
&\{h(\text{disruption plaque}, 0)\} \\
&\{\neg h(\text{disruption plaque}, 0)\} \\
&\{h(\text{erosion endo surface}, 0)\} \\
&\{\neg h(\text{erosion endo surface}, 0)\} \\
&\leftarrow h(\text{stemi}, 0) \\
&\leftarrow \neg h(\text{stemi}, \text{maxstep})
\end{align*}
\]

Here, we consider only \text{disruption plaque} and \text{erosion endo surface} to be exogenous since these are the only fluents that do not occur in the head of any rule. Since we need to retrieve the sequence of causes, it might be sufficient if only such fluents are considered to be exogenous. However, in general, we might need to consider all fluents to be initially exogenous (like in \( Q_1 \)). For \( \text{maxstep} = 1 \) and 2, \( (\Pi \cup I \cup Q_2)^t \) has no answer sets. For \( \text{maxstep} = 3 \), \( (\Pi \cup I \cup Q_2)^t \) is consistent, and the preferred answer sets contain \( h(\text{coronary occlusion}, 2) \), \( h(\text{occluding thrombus}, 1) \) and \( h(\text{disruption plaque}, 0) \), indicating that the most likely sequence of causes of STEMI is disruption of plaque followed by formation of a thrombus followed by a coronary occlusion. Note that even though some preferred answer sets might contain \( h(\text{erosion endo surface}, 0) \), those answer sets do not contain \( \text{apply}(r_{\text{thrombus}} 2(0)) \).

The second question provides an additional observation that disruption of plaque is not observed, and then asks for the likely cause of STEMI. To answer this question, we consider the query \( Q_3 \) obtained by adding the following rule to \( Q_2 \):

\[
\leftarrow h(\text{disruption plaque}, T)
\]

\(^{2}\)http://potassco.sourceforge.net/
This rule discards all sets in which disruption of plaque is true at some timepoint. Again, for maxstep 1 and 2, \((P \cup I \cup Q_3)^t\) has no answer sets. For maxstep = 3, \((P \cup I \cup Q_3)^t\) is consistent, and the preferred answer sets contain \(h\)\(\text{coronary occlusion}, 2\), \(h\)\(\text{occluding thrombus}, 1\) and \(h\)\(\text{erosion endo surface}, 0\), indicating that the most likely sequence of causes of STEMI is erosion of the endothelial surface followed by formation of a thrombus followed by a coronary occlusion. The reason for this result is as follows. Since we discard all sets in which disruption of plaque is true, any answer set containing \(\text{apply}(r_{\text{thrombus}}_1(T))\) for some timepoint \(T\) is not non-redundant. So, some of the non-redundant answer sets that contain the atoms \(h\)\(\text{erosion endo surface}, 0\) and \(\text{apply}(r_{\text{thrombus}}_2(0))\) become preferred answer sets.

The last question in the first set asks for the effects of erosion of the endothelial surface. This question can be answered by simply asserting \(h\)\(\text{erosion endo surface}, 0\) and increasing the value of maxstep to get a sequence of likely effects. Since the atom \(h\)\(\text{disruption plaque}, T\) does not hold for any timepoint \(T\), the non-redundant answer sets do not contain \(\text{apply}(r_{\text{thrombus}}_1(T))\) for any timepoint \(T\). As a result, some of the non-redundant answer sets that contain \(\text{apply}(r_{\text{thrombus}}_2(0))\) become preferred answer sets.

### 4.2 Answering Questions Related to the Second Set of Sentences

A simplified representation of the second set of sentences is given by the following program \(P\):

\[
\begin{align*}
\text{mechanical complication(mitral regurgitation)} \\
\text{mechanical complication(ventricular septal defect)} \\
\text{arrhythmia(ventricular fachycardia)} \\
\text{arrhythmia(ventricular fibrillation)} \\
\text{arrhythmia(atrioventricular block)} \\
\text{r_{hf}1(T)}: \{h(\text{heart failure, T + 1})\} \leftarrow h(\text{myocardial damage, T}) \\
\text{r_{hf}2(A, T)}: \{h(\text{heart failure, T + 1})\} \leftarrow h(A, T) \\
\text{r_{hf}3(M, T)}: \{h(\text{heart failure, T + 1})\} \leftarrow h(M, T) \\
\text{r_{arrhy}(A, T)}: \{h(A, T + 1)\} \leftarrow h(\text{ischaemia, T}) \\
\text{r_{scd}(A, T)}: \\
\quad \{h(\text{sudden cardiac death, T + 1})\} \leftarrow h(A, T), h(\text{acute ischaemic syndrome, T}) \\
\quad \text{prefer}(r_{hf}1(T), r_{hf}2(A, T)) \\
\quad \text{prefer}(r_{hf}1(T), r_{hf}3(M, T))
\end{align*}
\]

Here, \(A\) and \(M\) range over the extents of \text{arrhythmia} and \text{mechanical complication} respectively. In addition to these rules, we also have constraints to represent that a patient can have at most one form of arrhythmia or mechanical complication at any given
time. The first question can be answered using the following query $Q_1$:

\[
\begin{align*}
\{ & h(\text{myocardial damage}, 0) \} & \{ & \neg h(\text{myocardial damage}, 0) \} \\
\{ & h(M, 0) \} & \{ & \neg h(M, 0) \} \\
\{ & h(\text{ischaemia}, 0) \} & \{ & \neg h(\text{ischaemia}, 0) \} \\
\{ & h(\text{acute ischaemic syndrome}, 0) \} & \{ & \neg h(\text{acute ischaemic syndrome}, 0) \} \\
\langle & \neg h(\text{heart failure}, \text{maxstep}) & \rangle & \langle \neg h(\text{ventricular tachycardia}, 1) & \rangle & \\
\langle & h(\text{myocardial damage}, T) & \rangle & \langle h(M, T) & \rangle
\end{align*}
\]

The first 4 lines represent the exogeneity of fluents at the initial timepoint, and the rest of the lines represent the observations. For $\text{maxstep} = 2$, every preferred answer set of $(\Pi \cup I \cup Q_1)^t$ contains $h(\text{ischaemia}, 0)$ and one of $h(\text{ventricular tachycardia}, 1)$, $h(\text{ventricular fibrillation}, 1)$ and $h(\text{atrioventricular block}, 1)$, which indicates that the likely cause is ischaemia followed by arrhythmia.

The second question can be answered by using the query $Q_2$ that is obtained from $Q_1$ by replacing the observations (last 3 lines) with the following:

\[
\langle & \neg h(\text{sudden cardiac death}, \text{maxstep}) & \rangle
\]

For $\text{maxstep} = 2$, every preferred answer set of $(\Pi \cup I \cup Q_2)^t$ contains $h(\text{ischaemia}, 0)$, $h(\text{acute ischaemic syndrome}, 1)$, and one of $h(\text{ventricular tachycardia}, 1)$, $h(\text{ventricular fibrillation}, 1)$ and $h(\text{atrioventricular block}, 1)$, which indicates that the likely causes are acute ischaemic syndrome, and ischaemia followed by arrhythmia.

\section{Related Work and Conclusion}

There have been several approaches presented for handling preferences in ASP that allow users to explicitly specify the preferences among rules [6; 7; 8]. However, it appears that these approaches cannot be straightforwardly used for our purpose. For example, consider the program

\[
\Pi = (10) \cup \{ \langle s \rangle \} \cup \{ \langle \neg p \rangle \}.
\]

As discussed earlier, we expect $\{p, q, s\}$ to be one of the answer sets of the program. However, it is not B/W/D-preferred [9]. One can try using CR-Prolog by treating the rules $r_1 - r_4$ as cr-rules. However, this makes the program inconsistent. Also, treating the more preferred rules ($r_1, r_3$) as normal rules and the less preferred rules ($r_2, r_4$) as cr-rules results in the answer sets $\{p, r, s\}$ and $\{p, q, r, s\}$.

In this note, we showed how ASP can be used to represent and reason with preferences and uncertainty in dynamic domains. The approach we presented is primarily

\footnote{B-preferred, W-preferred and D-preferred answer sets are the preferred answer sets according to the approaches in [6], [7] and [8] respectively. Note that $\Pi$ needs to be rewritten in the respective languages to apply these approaches. Choice constructs can be eliminated using new atoms.}
aimed at answering questions regarding diagnosis and treatment of diseases and disorders. While the approach seems to be suitable for answering the types of questions considered in this paper, it still needs to be evaluated on a variety of other complex questions. As part of the future work, we plan to provide semantics for the language introduced in this paper and compare it with the existing approaches for representing preferences. We also plan to investigate the use of causal logic [10, 11] for answering questions similar to the ones considered in this paper.

References